Information acquisition, urgent production and welfare

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October, 2026

Introduction

- **Firms acquire demand information because production takes time:**
  - 90% of Walmart's largest suppliers subscribed to Luminate Charter for shopper and retail data.
  - Michelin used customer demand and inventory Kbrw data to schedule replenishment and adjust tire production.
  - Drees Homes uses Zonda's market data to plan construction before breaking ground.
- **Question:** How do private incentives to acquire demand information compare with its social value when production takes time?

Sources: Walmart (2024); Kbrw's Michelin case study; Zonda's Drees Homes case study (2026).

Preview

- **Question:** How do private incentives to acquire demand information compare with its social value when production takes time?
- **Model** with monopolistically competitive firms, taste shocks, information acquisition, and differences in costs of producing in advance and on the spot.
- **Results:**
  1. Distortion in production => distortion in information acquisition.
  2. Positive information externality: better information raises consumption utility => firms under-acquire information.
  3. Inducing socially optimal amount of information improves welfare by 1.07% in a calibrated model.

Roadmap

1. Environment.
2. Operating optimum:
   - firm operating problem;
   - optimal supply function;
   - optimal inventory choice.
3. Welfare and information under-acquisition.
4. Quantitative results and policy implications.

Environment

Preferences

A representative household consumes the unit mass of varieties $i \in [0,1]$.

CES consumption index

$$C_t = \left(\int_0^1 \theta_{i,t}^{1/\eta} q_{i,t}^{(\eta-1)/\eta} \mathrm{d}i \right)^{\eta/(\eta-1)}, \quad \eta > 1.$$

CRRA consumption utility

$$u_\gamma(C) = \begin{cases} \dfrac{C^{1-\gamma}-1}{1-\gamma}, & \gamma \neq 1, \\ \log C, & \gamma = 1, \end{cases} \qquad \gamma > 0.$$

Lifetime preferences

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t \left[u_\gamma(C_t) - L_t^R - \int_0^1 \nu_i L_{i,t}^U \mathrm{d}i \right], \qquad \beta \in (0,1), \quad \nu_i > 1.$$

Regular labor has unit marginal disutility; urgent labor has marginal disutility ν_i.

Technology and inventories

Firm $i \in [0,1]$ produces variety i and carries inventories $I_{i,t} \geq 0$, with depreciation rate δ_i.

The firm chooses a state-contingent quantity rule $q(\cdot)$ before demand is realized. $\hat{q}$ denotes quantity sold by the firm.

Production and inventories

$$\hat{q}_{i,t} \leq J_{i,t-1} + Y^U_{i,t},$$

$$Y^U_{i,t} = L^U_{i,t} \geq 0, \qquad J_{i,t-1} := (1-\delta_i) I_{i,t-1}.$$

Net inventory requirement after sales:

$$I_{i,t} - (J_{i,t-1} - \hat{q}_{i,t})^+.$$

> 0: new inventories required; < 0: returned inventories.

Inventory sector: one unit of regular labor produces one inventory unit $Y^R_t = L^R_t$; returned inventories are reallocated.

Uncertainty, information, and costs

Taste shocks and common history The log taste shock for each good follows the same AR(1) law with independent idiosyncratic innovations, so $\theta_{i,t}$ summarizes h^t for forecasting $\theta_{i,t+1}$.

$$\boldsymbol{\theta}_t := \{\theta_{i,t}\}_{i\in[0,1]}, \qquad h^t := (\boldsymbol{\theta}_0, \ldots, \boldsymbol{\theta}_t).$$

Prior: $\log \theta_{i,t+1} \mid \theta_{i,t} \sim N\left(\mu^-_{i,t}, (\sigma^-)^2\right), \qquad \sigma^- > 0,$

Signal: $s_{i,t} = \log \theta_{i,t+1} + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim N(0, \sigma^2_{\varepsilon,i,t}), \qquad \sigma_{i,t} \in (0, \sigma^-]$

Posterior: $\log \theta_{i,t+1} \mid \theta_{i,t}, s_{i,t} \sim N\left(\mu_{i,t}, \sigma^2_{i,t}\right).$

Information cost

$$\mathcal{I}_i(\sigma^-) = 0, \qquad \mathcal{I}_i'(\sigma) < 0, \qquad \mathcal{I}_i''(\sigma) \geq 0.$$

Smaller $\sigma_{i,t}$ means more precision and a higher cost; $\sigma_{i,t} = \sigma^-$ is the no-information limit.

Timing

Each date has three stages.

1. $\boldsymbol{\theta}_t$ is realized. The quantity rule chosen at $t-1$ determines realized sales, and inverse demand determines the realized price:

$$\hat{q}_{i,t} := q_{i,t-1}(\theta_{i,t}), \qquad \hat{p}_{i,t} := C_t^{1/\eta-\gamma}\theta_{i,t}^{1/\eta}\hat{q}_{i,t}^{-1/\eta}.$$

   Inventories and urgent production satisfy sales.

2. Firm i chooses $\sigma_{i,t}$ and observes private signal $s_{i,t}$ about $\theta_{i,t+1}$.

3. Conditional on $(\theta_{i,t}, s_{i,t})$, firm i chooses $I_{i,t}$ and the contingent quantity rule $q_{i,t}(\cdot)$ for date $t+1$.

▸ Markets

Operating decisions and inventory choice

Firm's operating problem

Given information precision, the dynamic operating problem reduces to a static problem after each signal $s_{i,t}$.

$$J_{i,t} := (1-\delta_i) I_{i,t}, \qquad \lambda_{i,t} := \frac{p_t^I}{\beta(1-\delta_i)} - p_{t+1}^I.$$

Choose the contingent quantity rule and usable inventories

$$\max_{q_{i,t}(\cdot)\geq 0,\, J_{i,t}\geq 0} \left\{ \mathbb{E}\left[(\hat{p}_{i,t+1} - p_{t+1}^I)\hat{q}_{i,t+1} - (\nu_i - p_{t+1}^I)(\hat{q}_{i,t+1} - J_{i,t})^+ \,\middle|\, \theta_{i,t}, s_{i,t} \right] - \lambda_{i,t} J_{i,t} \right\}.$$

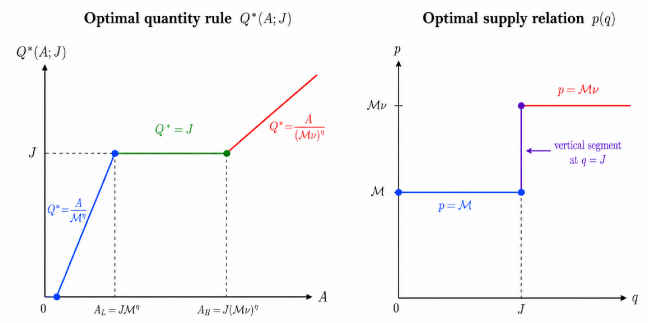
For each realized taste shock, the rule determines sales and inverse demand determines the price:

$$\hat{q}_{i,t+1} := q_{i,t}(\theta_{i,t+1}), \qquad \hat{p}_{i,t+1} = C_{t+1}^{1/\eta-\gamma} \theta_{i,t+1}^{1/\eta} \hat{q}_{i,t+1}^{-1/\eta}, \qquad \log \theta_{i,t+1} \mid \theta_{i,t}, s_{i,t} \sim N(\mu_{i,t}, \sigma_{i,t}^2).$$

Information costs are sunk when these operating choices are made.

▸ Full Bellman problem ▸ σ-equilibrium ▸ Welfare optimum ▸ Household problem

Optimal piecewise-linear supply function



$$\mathrm{MR}(Q) = \frac{1}{\mathcal{M}}\left(\frac{A}{Q}\right)^{1/\eta}, \qquad 1 \le \mathrm{MR}(J) \le \nu.$$

$A = C^{1-\eta\gamma}\theta$, $J = (1-\delta)I$, and $\mathcal{M} = \eta/(\eta-1)$, $p^I = 1$.

▸ Quantity rule

▸ Inventory choice

Welfare and information under-acquisition

Why is information valuable?

Optimal allocation

Given a fixed expected production, the optimal allocation across demand states equates marginal consumption values:

$$\frac{\theta^h}{q^h} = \frac{\theta^l}{q^l}.$$

Binding inventories

When inventories bind at J, the high-demand state has higher marginal consumption value:

$$\frac{\theta^h}{J} > \frac{\theta^l}{J}.$$

Shifting a fixed expected quantity toward that state raises consumption utility => information has positive consumption value.

- Note: for $\nu = 1$ quantities are allocated optimally => advance information has no operating value.

▸ Allocation problem

Why firms capture only part of the information value?

At fixed aggregate consumption, integrating inverse demand gives

$$\int_0^{\hat{q}_i} p_i(z)\,\mathrm{d}z = \mathcal{M} R_i, \qquad R_i = \hat{p}_i \hat{q}_i, \qquad \mathcal{M} = \frac{\eta}{\eta - 1} > 1.$$

- The firm captures only $1/\mathcal{M}$ of this gross consumption contribution while paying the full operating cost. It does not *fully* value the consumption gains from better information.

Sales subsidy

$X = \mathcal{M} - 1$ gives the firm $(1 + X)R_i = \mathcal{M} R_i$: aligns social and private incentives for information acquisition.

Production subsidy

Reimbursing $1 - 1/\mathcal{M}$ of operating costs induces the same operating choices at a given information profile, but only $1/\mathcal{M}$ of the sales subsidy's private information gain.

Under-acquisition of information

Type-wide experiment. A type g is a mass m_g of firms with the same primitives; $\mathcal{T}_g$ is the set of those firms. An information improvement lowers their common posterior uncertainty, $\mathrm{d}\sigma_g < 0$, at date t.

$$W^{\text{net}} := \mathbb{E}_t \sum\nolimits_{\tau=t}^{\infty} \beta^{\tau-t} \left[u_\gamma(C_\tau^*) - L_\tau^{R,*} - \int_0^1 \nu_i L_{i,\tau}^{U,*} \,\mathrm{d}i - \int_0^1 \mathcal{I}_i(\sigma_{i,\tau}) \,\mathrm{d}i \right].$$

For this experiment, define

$$\mathrm{d}W^{\text{net}} := \frac{\partial W^{\text{net}}}{\partial \sigma_{g,t}} \mathrm{d}\sigma_g, \quad U := \beta \mathbb{E}_t[u_\gamma(C_{t+1}^*)], \quad \mathrm{d}U := \frac{\partial U}{\partial \sigma_{g,t}} \mathrm{d}\sigma_g.$$

Information is under-acquired in the decentralized equilibrium

$$\mathrm{d}W^{\text{net}}\Big|_{\boldsymbol{\sigma}^p} = \left(1 - \frac{1}{\mathcal{M}}\right) \mathrm{d}U > 0,$$

where

$$\mathrm{d}U = -\frac{\beta \sigma_g}{\eta \gamma} \left[\int_{\mathcal{T}_g} \mathbb{E}_t[R_i \mathbf{1}_{\mathcal{A}_i}] \,\mathrm{d}i \right] \mathrm{d}\sigma_g > 0 \quad \text{for } \mathrm{d}\sigma_g < 0.$$

▸ Private versus social value ▸ Welfare-maximizing information choice

Other results and evidence

- **Urgency costs can raise decentralized welfare.** More costly urgent production can induce enough additional information to outweigh its direct resource cost.
  ▸ Welfare decomposition
- **Information subsidy.** Reimbursing $\zeta^* = 1/[1 + \gamma(\eta - 1)]$ of information costs aligns private and social marginal information incentives at the operating allocation. ▸ Policy details
- **Quantitative results.** Moving to the welfare-optimal information profile raises benchmark welfare by **1.07%**. ▸ Quantitative model

Takeaways

1. This paper characterizes optimal inventory decision and supply function under demand uncertainty and costly urgent production.

   Optimal supply function is piecewise linear.

2. Firms under-acquire information relative to welfare-maximizing choice; the quantitative welfare gain from correcting the wedge is economically meaningful.

Appendix

Firm's problem: full Bellman representation

Given a profile of posterior uncertainties $\boldsymbol{\sigma}$ and a state $x_{i,t} = (q_{i,t-1}(\cdot), I_{i,t-1}, \theta_{i,t})$, let $J_{i,t-1} := (1-\delta_i) I_{i,t-1}$. Firm i's net continuation value solves

$$V_i(x_{i,t}; \boldsymbol{\sigma}) = \mathbb{E}\left[\max_{q_{i,t}(\cdot) \geq 0,\, I_{i,t} \geq 0} \mathbb{E}\left[\pi^0_{i,t} - \mathcal{I}_i(\sigma_{i,t}) + \beta V_i(x_{i,t+1}; \boldsymbol{\sigma}) \mid \theta_{i,t}, s_{i,t} \right] \middle| \theta_{i,t} \right].$$

$$\pi^0_{i,t} := \hat{p}_{i,t} \hat{q}_{i,t} - p^I_t \left[I_{i,t} - (J_{i,t-1} - \hat{q}_{i,t})^+ \right] - \nu_i L^U_{i,t} \qquad \text{(operational profit)}$$

Realized outcomes

Quantity rule: $\hat{q}_{i,t} = q_{i,t-1}(\theta_{i,t})$,

Inverse demand: $\hat{p}_{i,t} = C_t^{1/\eta - \gamma} \theta_{i,t}^{1/\eta} \hat{q}_{i,t}^{-1/\eta}$,

Urgent production: $L^U_{i,t} = (\hat{q}_{i,t} - J_{i,t-1})^+$,

Net inventory order: $I_{i,t} - (J_{i,t-1} - \hat{q}_{i,t})^+$.

Information

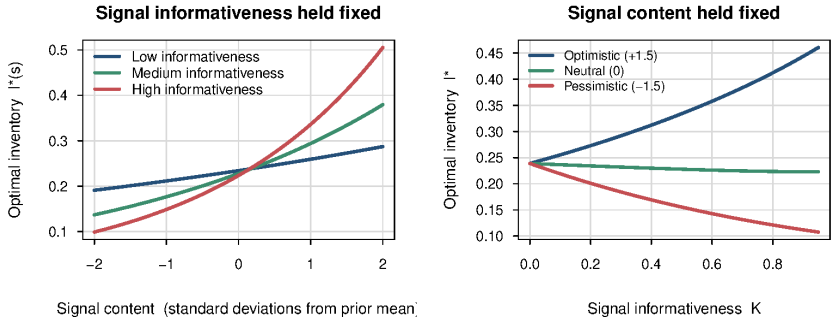
Prior: $\log \theta_{i,t+1} \mid \theta_{i,t} \sim N(\mu^-_{i,t}, (\sigma^-)^2)$,

Posterior: $\log \theta_{i,t+1} \mid \theta_{i,t}, s_{i,t} \sim N(\mu_{i,t}, \sigma^2_{i,t})$,

Cost: $\mathcal{I}_i(\sigma_{i,t})$ is strictly decreasing and convex.

▸ σ-equilibrium ▸ Decentralized and welfare optimum ◂ Back to static problem

Optimal inventory choice (and information)



$$\log I^*(K,s) = \underbrace{\mu^- + K(s-\mu^-)}_{\text{posterior-mean channel}} + \underbrace{\sigma(K)a^*(\sigma(K))}_{\text{cutoff channel}} - \eta \log(\mathcal{M} p^I_{t+1}) - \log(1-\delta), \quad a := \frac{\log A_L - \mu^A_{i,t}}{\sigma_{i,t}}.$$

- **Posterior-mean channel:** greater uncertainty makes firm put less weight on the signal.
- **Cutoff channel:** $a^*(\sigma) + \sigma \frac{da^*(\sigma)}{d\sigma}$. $\frac{da^*(\sigma)}{d\sigma} > 0$: greater uncertainty pushes more high-demand states into urgent production region. Increase cutoff a^* to restore optimality.

Inventory choice | Back to supply plots

The allocation benchmark: match quantities to tastes

Fix one good's conditional distribution of $\theta_{i,t+1}$ and its expected sales $\bar{q} > 0$. Suppress $i, t+1$ and write $q(\theta)$ for realized sales $\hat{q}_{i,t+1}$.

Allocation problem

$$\max_{q(\cdot)\geq 0} \mathbb{E}_t\left[\theta^{1/\eta} q(\theta)^{(\eta-1)/\eta}\right] \quad \text{s.t.} \quad \mathbb{E}_t[q(\theta)] = \bar{q}.$$

Optimal allocation

$$\underbrace{S\left(\frac{\theta}{q(\theta)}\right)^{1/\eta}}_{p(\theta)} = \zeta \quad \Longrightarrow \quad q^*(\theta) = \frac{\bar{q}\,\theta}{\mathbb{E}_t[\theta]}$$

Marginal consumption values are equalized:

$$\frac{q^h}{q^\ell} = \frac{\theta^h}{\theta^\ell}.$$

Two-state transfer

For two states with probabilities $w_h, w_\ell > 0$, shift ε expected units from low to high demand:

$$\mathrm{d}q^h = \frac{\varepsilon}{w_h}, \quad \mathrm{d}q^\ell = -\frac{\varepsilon}{w_\ell}, \quad \mathrm{d}\mathbb{E}_t[q] = 0, \qquad \mathrm{d}U = (p^h - p^\ell)\varepsilon.$$

At $q^h = q^\ell = J$, this gain is positive: high demand has higher marginal value.

With J fixed, raising high-state sales above J requires urgent production. Better information changes inventories before demand is realized; it need not hold expected sales or production costs fixed.

◂ Back to allocation intuition

Welfare-maximizing information choice given markups

At state x_t and after every common history, the planner chooses the history-contingent information profile:

$$\widetilde{\boldsymbol{\sigma}} \in \arg\max_{\boldsymbol{\sigma}} \; W^*(x_t;\boldsymbol{\sigma}) \qquad \text{subject to} \qquad 0 < \sigma_{i,\tau}(h^\tau) \leq \sigma^- \quad \forall i,\ \tau \geq t,\ h^\tau,$$

where

$$W^*(x_t;\boldsymbol{\sigma}) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[u_\gamma(C_\tau^*) - L_\tau^{R,*} - \int_0^1 \nu_i L_{i,\tau}^{U,*} \,\mathrm{d}i - \int_0^1 \mathcal{I}_i(\sigma_{i,\tau}) \,\mathrm{d}i \right].$$

Constrained information planner

For every candidate $\boldsymbol{\sigma}$, starred allocations come from the corresponding $\boldsymbol{\sigma}$-equilibrium. The planner chooses only information. Firms continue to choose inventories and supply functions, and prices retain the markup $\mathcal{M}$.

Thus, the planner compares information profiles given the same distorted operating behavior as in the decentralized economy.

Back to under-acquisition

Markets and the returnable-inventory sector

Take the regular wage as numeraire, $w_t^R = 1$. Variety and urgent-labor markets clear at

$$q_{i,t} = \hat{q}_{i,t}, \qquad L_{i,t}^U = (\hat{q}_{i,t} - J_{i,t-1})^+, \qquad w_{i,t}^U = \nu_i \text{ when urgent labor is used.}$$

Returnable-inventory market and price

The competitive sector reallocates returned units and produces any remaining shortfall one-for-one with regular labor:

$$Y_t^R = L_t^R = \int_0^1 \left[I_{i,t} - (J_{i,t-1} - \hat{q}_{i,t})^+ \right] \mathrm{d}i \geq 0.$$

Zero profit and unit production cost imply

$$0 < p_t^I \leq 1, \qquad L_t^R > 0 \implies p_t^I = 1, \qquad p_t^I < 1 \implies L_t^R = 0.$$

Firms are owned by the household and profits are rebated lump-sum; its budget binds at

$$\int_0^1 p_{i,t} q_{i,t} \mathrm{d}i = L_t^R + \int_0^1 w_{i,t}^U L_{i,t}^U \mathrm{d}i + \int_0^1 \pi_{i,t}^0 \mathrm{d}i.$$

Back to timing

A $\boldsymbol{\sigma}$-equilibrium fixes posterior uncertainty

Let

$$\boldsymbol{\sigma} := \{\sigma_{i,t}(\cdot)\}_{i\in[0,1],\, t\geq 0},$$

where each $\sigma_{i,t}(\cdot)$ maps date-t common histories into $(0, \sigma^{-}]$. Define pre-signal gross operating value by

$$\Pi_i^0(x_{i,t}; \boldsymbol{\sigma}) := \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi_{i,\tau}^0.$$

For a fixed profile $\boldsymbol{\sigma}$, equilibrium requires after every common history:

1. The household maximizes preferences subject to its budget constraint.
2. Each firm's urgent-production policy and its signal-contingent inventory and supply-function policies maximize Π_i^0, subject to technology and posterior beliefs, taking aggregate states, prices, and wages as given.
3. Market prices and sales satisfy household optimality and the inherited supply function; variety, labor, and inventory markets clear; the inventory-sector pricing rule holds; and

$$x_{i,t+1} = (f_{i,t}, l_{i,t}, h^{t+1}), \qquad h^{t+1} = (h^t, \boldsymbol{\theta}_{t+1}).$$

An asterisk denotes the corresponding $\boldsymbol{\sigma}$-equilibrium object.

◂ Back to firm problem

A type is a positive mass of firms with the same primitives

Partition firms into G types, indexed by $g = 1, \ldots, G$.

Type g

Type g has population mass $m_g > 0$ and common primitives

$$(\delta_i, \nu_i, \mathcal{I}_i(\cdot)) = (\delta_g, \nu_g, \mathcal{I}_g(\cdot)), \qquad i \in g,$$

with $\sum_{g=1}^{G} m_g = 1$. The symmetric economy is the special case $G = 1$.

The notation $\sigma_{g,t}$ denotes the posterior standard deviation common to type-g firms. Changing it moves the entire type, so

$$\frac{\partial \bar{V}_t^0}{\partial \sigma_{g,t}} = m_g \frac{\partial \bar{V}_{g,t}^0}{\partial \sigma_{g,t}}.$$

- Lower $\sigma_{g,t}$ means more information for the entire type.
- An individual firm is atomistic; type g has positive mass and moves aggregates.

Back to under-acquisition

Decentralized equilibrium and welfare optimum

Decentralized equilibrium

A decentralized equilibrium is a $\boldsymbol{\sigma}^P$-equilibrium in which each firm's history-contingent uncertainty policy $\{\sigma^P_{i,t}(\cdot)\}_{t\geq 0}$ maximizes

$$V_i(x_{i,t};\boldsymbol{\sigma}^P) = \Pi_i^{0,*}(x_{i,t};\boldsymbol{\sigma}^P) - \mathcal{K}_{i,t}(\boldsymbol{\sigma}^P)$$

after every common history. An atomistic firm takes aggregate allocations, prices, and other firms' uncertainty policies as given.

Welfare accounting and optimum

For any feasible profile $\boldsymbol{\sigma}$,

$$W^*(x_t;\boldsymbol{\sigma}) = CS^*(x_t;\boldsymbol{\sigma}) + \int_0^1 V_i(x_{i,t};\boldsymbol{\sigma})\mathrm{d}i.$$

A welfare-optimal information profile maximizes $W^*(x_t;\boldsymbol{\sigma})$ over feasible history-contingent profiles after every common history.

◂ Back to firm problem

Private and social information values differ

For a type-wide change in posterior uncertainty $\sigma_{g,t}$,

$$\frac{\partial W^*}{\partial \sigma_{g,t}} = \frac{\partial CS^*}{\partial \sigma_{g,t}} + \frac{\partial}{\partial \sigma_{g,t}} \int_0^1 \Pi_i^{0,*} \mathrm{d}i - m_g \frac{\partial \mathcal{K}_g}{\partial \sigma_{g,t}}.$$

Private information choice

An atomistic firm holds the common demand shifter fixed and equates its direct operating-profit effect to the marginal information-cost saving:

$$\frac{\partial \Pi_g^{0,*}}{\partial \sigma_{g,t}} = \frac{\partial \mathcal{K}_g}{\partial \sigma_{g,t}}.$$

- A type-wide change also changes aggregate consumption and the common CES demand shifter.
- The firm does not internalize the resulting effects on consumers or other firms.

◂ Back to firm problem ◂ Back to main result

Household problem and CES demand

The household solves

$$\max_{\{q_{i,t}, L_t^R, L_{i,t}^U\}} u_\gamma(C_t) - L_t^R - \int_0^1 \nu_i L_{i,t}^U \mathrm{d}i$$

subject to its period budget, where

$$C_t = \left(\int_0^1 \theta_{i,t}^{1/\eta} q_{i,t}^{(\eta-1)/\eta} \mathrm{d}i \right)^{\eta/(\eta-1)}.$$

Inverse demand

$$p_{i,t} = C_t^{1/\eta - \gamma} \theta_{i,t}^{1/\eta} q_{i,t}^{-1/\eta}.$$

The regular wage is the numeraire, $w_t^R = 1$. Urgent labor is supplied perfectly elastically at marginal disutility ν_i, so $w_{i,t}^U = \nu_i$.

Back to firm problem

Quantity rule and supply relation

Let $A := C^{1-\eta\gamma}\theta$, $c := p^I_{t+1}$, and $\mathcal{M} := \eta/(\eta-1)$. For fixed available inventories J,

Optimal quantity rule

$$Q^*(A;J) = \begin{cases} \dfrac{A}{(\mathcal{M}c)^\eta}, & A < J(\mathcal{M}c)^\eta, \\ J, & J(\mathcal{M}c)^\eta \le A \le J(\mathcal{M}\nu)^\eta, \\ \dfrac{A}{(\mathcal{M}\nu)^\eta}, & A > J(\mathcal{M}\nu)^\eta. \end{cases}$$

Implied supply relation

$$p(q) = \begin{cases} \mathcal{M}c, & q < J, \\ \text{vertical segment at } q = J, & p \in [\mathcal{M}c, \mathcal{M}\nu], \\ \mathcal{M}\nu, & q > J. \end{cases}$$

◂ Back to supply slide

Inventory choice: solving for a^*

For fixed posterior uncertainty, set

$$\lambda_{i,t}^{I} := \frac{p_t^I}{(1-\delta_i)\beta} - p_{t+1}^I, \quad c := p_{t+1}^I, \quad \tau_{i,t} := \frac{\sigma_{i,t}}{\eta}, \quad d_{i,t} := \frac{\eta \log(\nu_i/c)}{\sigma_{i,t}}.$$

With ϕ and Φ denoting the standard-normal density and cdf, define

$$G_{i,t}(a) := \int_a^{a+d_{i,t}} \left(e^{\tau_{i,t}(z-a)} - 1\right) \phi(z)\mathrm{d}z + \left(\frac{\nu_i}{c} - 1\right) \left[1 - \Phi(a + d_{i,t})\right].$$

The lower standardized cutoff solves the scalar equation

$$\boxed{G_{i,t}(a_{i,t}^*) = \frac{\lambda_{i,t}^I}{c}}.$$

It is unique because

$$G_{i,t}'(a) < 0, \qquad G_{i,t}(-\infty) = \frac{\nu_i}{c} - 1, \qquad G_{i,t}(+\infty) = 0,$$

provided that $0 < \lambda_{i,t}^I/c < \nu_i/c - 1$.

Back to inventory choice

Inventory choice

Suppose

$$\log A_{i,t+1} \mid \theta_{i,t}, s_{i,t} \sim N\left(\mu^A_{i,t}, \sigma^2_{i,t}\right), \qquad c := p^I_{t+1}, \qquad \lambda^I_{i,t} := \frac{p^I_t}{(1-\delta_i)\beta} - p^I_{t+1}.$$

For a candidate after-depreciation inventory position J, define the lower and upper standardized cutoffs by

$$a = \frac{\log[J(\mathcal{M}c)^\eta] - \mu^A_{i,t}}{\sigma_{i,t}}, \qquad b = a + \frac{\eta \log(\nu_i/c)}{\sigma_{i,t}}.$$

Let $a^*_{i,t}$ denote the unique lower cutoff obtained from the linked scalar equation.

Optimal inventories

$$J^*_{i,t} = \frac{\exp(\mu^A_{i,t} + \sigma_{i,t} a^*_{i,t})}{(\mathcal{M}c)^\eta}, \qquad I^*_{i,t} = \frac{J^*_{i,t}}{1-\delta_i}.$$

- Solve for $a^*_{i,t}$ before using the expression for $J^*_{i,t}$.
- The cutoff does not depend on $\mu^A_{i,t}$; the posterior mean shifts log inventories one for one.
- Prices enter through $\lambda^I_{i,t}/c$ and the urgency wedge $\nu_i/c - 1$.

Solving for a^* | Back to supply slide

Profit, revenue, and the aggregate CES shifter

In the active-production benchmark, normalize the common demand shifter to one. At the operating optimum,

$$\bar{V}^0_{i,t} = \frac{1}{\eta}\mathbb{E}_t[\bar{R}_{i,t+1}], \qquad \partial_{\sigma_{i,t}} \bar{V}^0_{i,t} \leq 0.$$

General-equilibrium value is

$$V^0_{i,t} = \left(\eta \bar{V}^0_t\right)^{\frac{1-\eta\gamma}{\gamma(\eta-1)}} \bar{V}^0_{i,t}.$$

For a type-wide uncertainty change,

Aggregate operating-profit response

$$\frac{\partial V^0_t}{\partial \sigma_{g,t}} = \frac{V^0_t}{\bar{V}^0_t} \frac{1-\gamma}{\gamma(\eta-1)} \frac{\partial \bar{V}^0_t}{\partial \sigma_{g,t}}.$$

Because $\partial_{\sigma_g} \bar{V}^0_t < 0$, aggregate profit rises with uncertainty when $\gamma > 1$, falls when $0 < \gamma < 1$, and is locally unchanged under log utility.

◂ Back to main result

Urgency costs can raise decentralized welfare

Let $\boldsymbol{\sigma}^p(\nu_{t+1})$ denote the private information profile chosen at date t.

Total effect of tomorrow's urgency cost

$$\frac{\mathrm{d}W^*}{\mathrm{d}\nu_{t+1}} = \underbrace{W^*_{\nu_{t+1}}}_{\text{direct resource-cost effect } <0} + \underbrace{\sum_g W^*_{\sigma_{g,t}} \frac{\mathrm{d}\sigma^p_{g,t}}{\mathrm{d}\nu_{t+1}}}_{\text{induced information effect}} .$$

If higher urgency costs induce more information,

$$\frac{\mathrm{d}\sigma^p_{g,t}}{\mathrm{d}\nu_{t+1}} \leq 0, \qquad W^*_{\sigma_{g,t}} < 0,$$

so the induced information effect is nonnegative.

- Welfare rises only if this indirect benefit dominates the direct cost.
- Date-t information responds to ν_{t+1}, not to the current urgency cost.

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Subsidizing information acquisition

Without a sales subsidy, the government reimburses a fraction ζ of information expenditure. This leaves operating choices unchanged at a given information profile.

Marginal conditions for an interior choice

$$\text{Private:} \quad b_g = (1-\zeta) m_g \mathcal{I}_g'(\sigma_g^\zeta),$$

$$\text{Planner:} \quad \left[1 + \frac{1}{\gamma(\eta-1)}\right] b_g = m_g \mathcal{I}_g'(\sigma_g^\zeta).$$

$$\boxed{\zeta^* = \frac{1}{1+\gamma(\eta-1)}}$$

This aligns information incentives given monopolistic operating behavior; it does not remove the operating markup.

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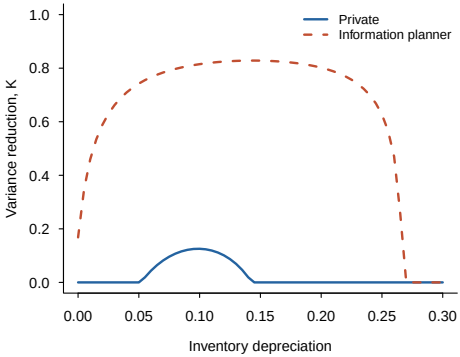
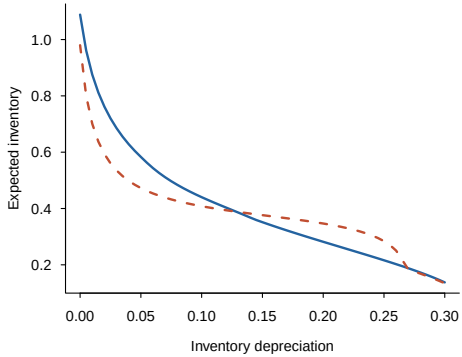
The quantitative model preserves the mechanism





Private firms acquire no information for 70.5% of types. Mean variance reduction rises from 2.6% privately to 64.3% under the planner; the one-time benchmark welfare gain is **1.07%**.

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Forces behind the information distortion

Case (change from paper benchmark)	Wedge	$\overline{K}^{p}$	$\overline{K}^{\mathrm{pl}}$	No-info. mass^{p}	Urgency premium^{p}	Welfare gain (%)
Paper benchmark: $(\eta, \gamma, \nu_g, (\sigma^-)^2, \kappa) = (1.6, 1, 1.5, 0.52, 0.04)$	1.667	0.026	0.643	0.705	2.503	**1.0681**
No urgent margin: ν_g: $1.5 \to 1$	1.667	0.000	0.000	1.000	0.000	0.0000
Cheaper information: κ: $0.04 \to 0.005$	1.667	0.933	0.980	0.000	0.179	0.1595
Moderately cheaper information: κ: $0.04 \to 0.02$	1.667	0.493	0.868	0.197	1.535	0.7508
More costly information: κ: $0.04 \to 0.08$	1.667	0.000	0.221	1.000	2.545	0.1359
Lower prior uncertainty: $(\sigma^-)^2$: $0.52 \to 0.26$, μ^-: $-0.13 \to 0$	1.667	0.000	0.427	1.000	1.393	0.3260
Smaller wedge: (η, γ): $(1.6, 1) \to (2.5, 1.5)$	0.444	0.430	0.612	0.230	1.798	0.1562

The wedge is $1/[\gamma(\eta - 1)]$. $K_g = 1 - \sigma_g^2/(\sigma^-)^2$ is the share of prior variance removed, and $\overline{K} = \sum_g m_g K_g$. The no-information mass is the share of types with $K_g = 0$. The urgency premium is $(\nu_g - 1)$ times urgent output divided by revenue. Each row changes only the parameters shown and re-solves the private equilibrium and the planner's information problem.

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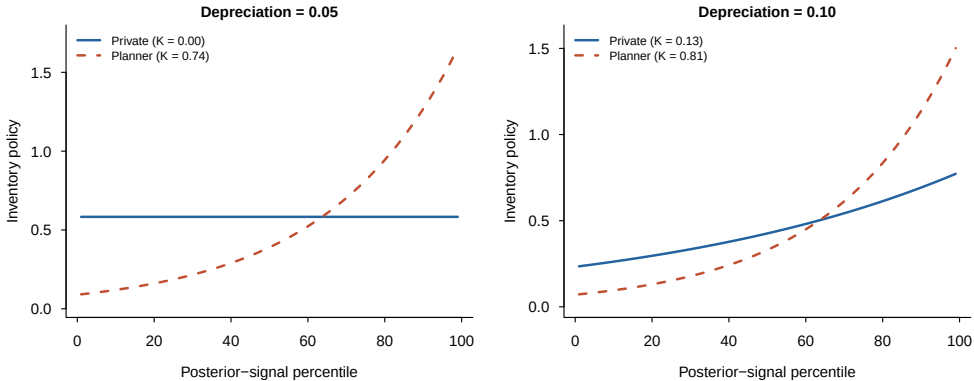
Information makes inventory policies more responsive



The welfare optimum uses more informative signals and therefore varies inventories more strongly with expected demand.

Data: management guidance and quarterly fundamentals

I/B/E/S Guidance

- Management sales guidance at the firm level.
- 13,108 firm-quarters and 1,219 U.S. firms.
- Guidance dates: 2018–2024; realized fiscal quarters through 2025Q3.
- Median horizon from announcement to quarter end: 57 days.

Compustat Quarterly

- Sales, COGS, inventories, assets, receivables, payables, cash, debt, CAPEX, and R&D.
- NAICS, SIC, and GICS industry controls.

Empirical objects:

$$FE_{it} = \log(\text{realized sales}_{it}) - \log(\text{guidance midpoint}_{it}).$$

Positive *FE* means realized sales exceed guidance. Forecast-error variance proxies for imprecision; inventories and COGS capture advance and urgent adjustment.

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Empirical evidence: information, inventories, and costly adjustment

Panel A. Information and inventory holdings

Object	Regressor	Outcome	Coef.	Sample
Expected sales	log guided sales$_{t+1}$	log I_t	0.214***	4,097
Demand surprise	FE_t	$\Delta \log I_t$	0.159**	4,400
Forecast imprecision	log Var(FE)	Avg. log I	0.029*	587 firms

Panel B. Costly urgent-production margin

Object	Regressor	Outcome	Coef.	Sample
Underestimated demand	$\max(FE_t, 0)$	$\Delta(\mathrm{COGS/Sales})_t$	0.044*	6,792
Overestimated demand	$\max(-FE_t, 0)$	$\Delta(\mathrm{COGS/Sales})_t$	-0.028	6,792

These are conditional correlations, not causal estimates. Back